

A Multi-length Bunches Design for Electron Storage Rings with Odd Buckets

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Abstract

A scheme with two superconducting RF cavities (sc-cavities) is designed to upgrade electron storage rings with odd buckets into multi-length bunches. In this paper, Hefei Light Source II (HLS II) is given as an example for odd buckets. In accordance with 45 buckets, which is multiples of 3, three different length of bunches generated simultaneously is proposed in the presently applied user optics. The final result is to, without low- α optics, fill HLS II with long bunches of 50 ps length, medium bunches of 23 ps and short bunches of 6 ps. Each third buckets can be filled with short bunches, of which the current limit is up to 6.6 mA, more than 60 times the value of low- α mode. Moreover, particles tracking about beam dynamics performed by ELEGANT and calculations about beam

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instabilities are presented in this paper.

Keywords: Multi-length, odd buckets, superconducting cavities, HLS

1. Introduction

An increasing interest in short x-ray pulses requires short electron bunches in storage rings. Over the years, the traditional way to get short bunches is to decrease the momentum compaction factor by low- α optics [1]. However, the
5 average current per bunch will decrease to the order of μA magnitude with low- α , limited by beam instability and microbunching due to coherent synchrotron radiation and other collective effects [6].

BESSY II presented an idea to produce long and short bunches alternatively in their storage ring: with two superconducting cavities, 3rd harmonic and
10 3.5th harmonic of the fundamental RF cavity, the voltage gradient produced by two sc-cavities could add up at even points for bunches focusing to get short bunches, and cancel each other at odd points to get long bunches [2]. The new method greatly improved the capacity of storing current in the ring. Nonetheless, BESSY II is filled with 400 buckets, which is the multiples of
15 2, leading to the result of choose the second sc-cavity as a $1/2$ times higher harmonic one. What if the ring is with odd buckets, such as multiples of 3, 5, 7? In this paper, we find the problem could be solved by choosing the frequency of the second sc-cavity (sub harmonic cavity) in a different way. For a ring, whose minimum common factor is k ($k > 1$), the frequency of the sub harmonic
20 cavity could be chosen $1/k$ times higher than harmonic one. Several typical situations are listed in Table.1, in which N_i ($i = 1, 2, 3, 4$) is chosen accord to the voltage and frequency of original cavity in different rings.

In addition, multi-cell structure is required for BESSY II and also other rings to achieve the 20 MV voltage of new sc-cavities because of the high original
25 cavity voltage, and multi-cell structure may lead to some HOM problems. So, if we could find a ring with low voltage, the HOM problems may be much simpler.

In this paper, we present a new scheme for HLS II, it is a good example

Table 1: Harmonic Number with Different Buckets Number

Common factor	Harmonic	Sub harmonic
2	N_1	$N_1 + 1/2$
3	N_2	$N_2 + 1/3$
5	N_3	$N_3 + 1/5$
7	N_4	$N_4 + 1/7$

to solve the odd buckets problem, and make the HOM problems easy at the same time. An idea with two sc-cavities whose voltages are around 2 MV and frequencies are around 1.3 GHz, is designed for HLS II. A 1/3 times higher harmonic sc-cavity is expected to be applied in HLS II, and with careful phase tuning, medium, long and short bunches are generated simultaneously. Moreover, because of the low voltage and frequency of fundamental cavity, it is easy to increase the voltage gradient in the ring by 100 times, and single-cell or two-cell structure is enough for two sc-cavities.

2. The Lattice of HLS II Storage Ring

Hefei Light Source II [3] is an 800 MeV electron storage ring, whose main parameters are given in Table.2. The lattice structure of HLS II was chosen as a DBA with 4 periods, and 8 straight sections which include four 4.0 m long straight sections and four 2.3 m short chromatic straight sections. In each half cell, there are four quadrupoles and four combined function sextupoles. The magnet layout of ring is shown in Fig.1 and Fig.2.

There are two operation modes in HLS II, Mode A is an achromatic mode whose dispersion in the long straight sections is zero, and Mode B is a distributed dispersion mode whose emittance is smaller than Mode A. In this paper, we choose the achromatic mode (Mode A) to make simulation. The main parameters of HLS II are summarized in Table.3, and Fig.3 shows the β and dispersion function per cell.

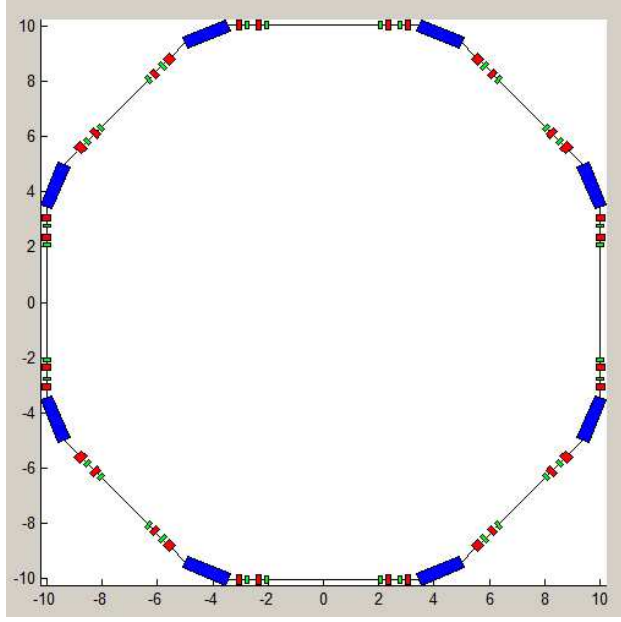


Figure 1: Magnet layout of HLS II. Bends are in blue rectangles, quadrupoles are in red rectangles and sextupoles are in green rectangles. The HLSII ring is a typical octagonal structure.

3. The Medium-Long-Short Bunches Scheme

50 In an electron storage ring, the equilibrium bunch length is [7]

$$\sigma_s = \frac{c \delta_\varepsilon}{2\pi} \sqrt{\frac{4\pi^2 \alpha E_0}{ce f_{rev} V'}} \quad (1)$$

here f_{rev} is the revolution frequency, α is the momentum compaction factor, V' is the voltage gradient, δ_ε is the equilibrium energy spread. From this relation, the bunch length can be shortened by decreasing momentum compaction factor or increasing the voltage gradient for $\sigma_s \propto \sqrt{\alpha/V'}$. In this section, we just focus
 55 on the method of increasing the voltage gradient.

The original cavity is rf-frequency $f_0 = 0.204$ GHz, voltage $V_0 = 0.25$ MV and phase $\psi_{s0} = 3.0747$ rad. It is used to replenish the energy loss by synchrotron radiation, and the frequency also leads to the fill pattern of 45 buckets. Because of the high gradient for bunch focusing, two superconducting cavities are required
 60 here. The first one is a sixth harmonic cavity $f_1 = 1.2240$ GHz ($f_1 = 6f_0$),

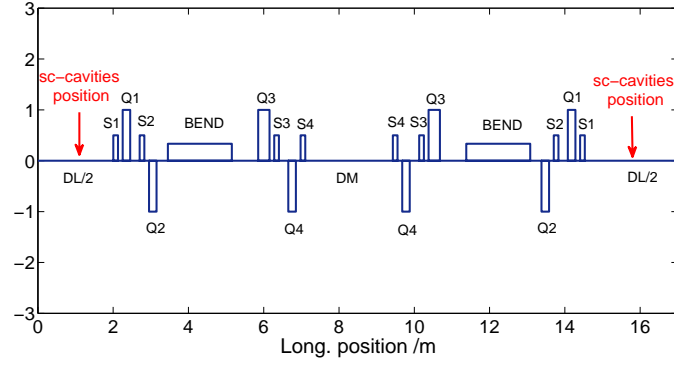


Figure 2: The lattice structure per cell in HLS II

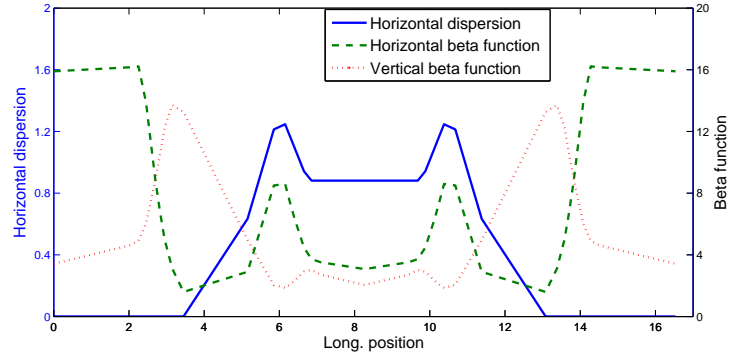


Figure 3: β and dispersion function of HLS II. The new superconducting cavities are required to be located at the zero dispersion position shown in Figure 2.

Table 2: General Parameters of HLS II.

Nominal energy	800 MeV
Circumference	66.13 m
Number of cells	4
Number of dipoles	8
Bending radius	2.1645 m
RF frequency	0.204 GHz
Energy spread	0.00047
Emittance	36.38 nm · rad
Beam current	> 300 mA
Momentum compaction	0.0205
Damping time (τ_x, τ_y, τ_s)	(20.00, 21.08, 10.84) ms
Tunes(ν_x, ν_y)	(4.44, 2.80)
Nature chromaticity(ξ_x, ξ_y)	(-9.89, -4.66)
Energy loss per turn	16.74 keV

and considering the 45 buckets, the second cavity is chosen as $(f_2 = (6 + \frac{1}{3})f_0)$, a 1/3 times higher harmonic in rf-frequency. The two sc-cavities can devote to three kinds of bunches.

In addition to the rf-frequency, finite voltage and phase of sc-cavities must
65 be chosen for particle acceleration and stable buckets. The sum voltage gradient at longitudinal distance is

$$V'_{\text{sum}} = \frac{2\pi}{c} \cdot [fv]_{\text{focusing}} \quad (2)$$

and $[fv]_{\text{focusing}}$ is defined as a rf-focusing parameter which directly dominates the bunch length

$$[fv]_{\text{focusing}} = \left[f_0 V_0 \cos\left(\frac{2\pi f_0}{c}s + \psi_{s0}\right) + f_1 V_1 \cos\left(\frac{2\pi f_1}{c}s + \psi_{s1}\right) + f_2 V_2 \cos\left(\frac{2\pi f_2}{c}s + \psi_{s2}\right) \right] \quad (3)$$

here $\psi_{s0}, \psi_{s1}, \psi_{s2}$ is the phase of original, harmonic and sub harmonic cavity.

Table 3: The Main parameters of Lattice(Half cell)

Start point	Midpoint of DL
DL/2	2.003175 m
S1	0.00 m ⁻³
Q1	3.8807 m ⁻²
S2	0.00 m ⁻³
Q2	-3.2031 m ⁻²
DBQ1	0.30 m
B	1.7 m / 1.2336 T
DBQ2	0.70 m
Q3	3.7871 m ⁻²
S3	49.36 m ⁻³
Q4	-3.3874 m ⁻²
S4	-79.07 m ⁻³
DM/2	1.163175 m
Symmetric point	Midpoint of DL

70 For a proper canceling to get long bunches, frequencies and voltage amplitudes are required to be $f_1 V_1 = f_2 V_2$ [2]. Although higher voltage could shorten bunches more, taking the HOM problem and cavities design into account, voltages are chosen as $V_1=1.9$ MV and $V_2=1.8$ MV.

To get ψ_{s1} and ψ_{s2} , 45 buckets are divided into three groups: $\text{Bucket}_{(3m)}$,
75 $\text{Bucket}_{(3m+1)}$, $\text{Bucket}_{(3m+2)}$, ($m = 0, 1, 2, 3, \dots, 14$). Two conditions are made here to determine the phase for cavities. First, only original cavity is used for energy recovery because of the expensive cooling system for superconducting cavities. Second, voltage gradient produced by two cavities are expected to cancel each other at the position of $\text{Bucket}_{(3m+1)}$ to get long bunches. Two
80 conditions can be summarized as follows

$$\begin{aligned} \text{condition 1} \quad & V_1 \sin(\psi_{s1}) + V_2 \sin(\psi_{s2}) = 0 \\ \text{condition 2} \quad & \psi_{s1} + \psi_{s2} = \frac{1}{3}\pi + k\pi \end{aligned} \tag{4}$$

here k is an integer.

By solving the conditions, phases are chosen as $\psi_{s1} = 2.1412$ rad and $\psi_{s2} = -1.0940$ rad. The main parameters of three cavities are given in Table.4. And two sc-cavities positions require zero dispersion to avoid bunch lengthening by coupling effects [9], shown in Fig.2.

Table 4: The Main Parameters of Three Cavities

Cavities	Harmonic	Frequency	Voltage	Phase
	Number	f (GHz)	V (MV)	ψ_s
Original	1	0.204	0.25	176.17°
Harmonic	6	1.2240	1.9	122.68°
Sub Harmonic	$6\frac{1}{3}$	1.2920	1.8	-62.68°

85

After phase tuning, the voltage gradient produced by the two sc-cavities cancel each other at the $\text{Bucket}_{(3m+1)}$ to get long bunches, add up at the $\text{Bucket}_{(3m+2)}$ to get short ones, and $\text{Bucket}_{(3m)}$ can be filled with medium bunches.

90 Fig.4 shows the sum voltage of three cavities as a function of the longitudinal

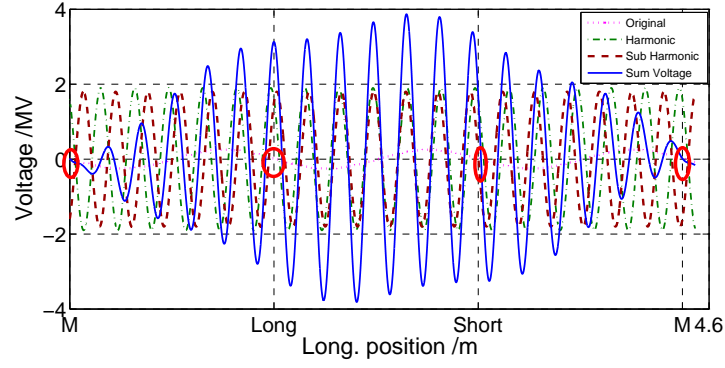


Figure 4: Voltage of Three Cavities. The sum voltage in MV as a function of the longitudinal distance in m is shown in blue. Voltages of the original, harmonic and sub harmonic cavities are shown in purple, green and red.

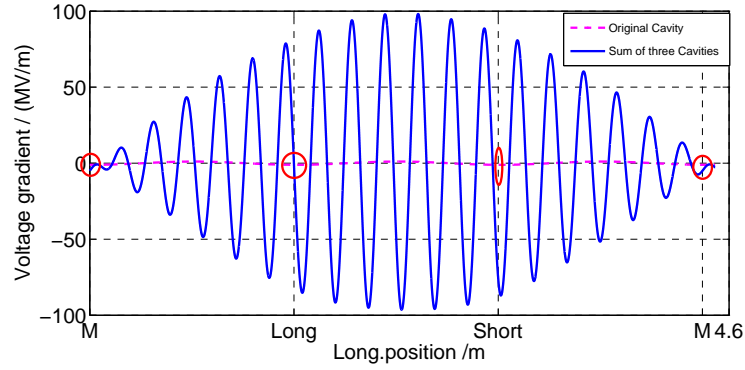


Figure 5: The Sum Voltage Gradient of Three Cavities. The sum voltage gradient in MV/m as a function of the longitudinal distance in m is shown in blue. The voltage gradient of the original cavity is shown in purple.

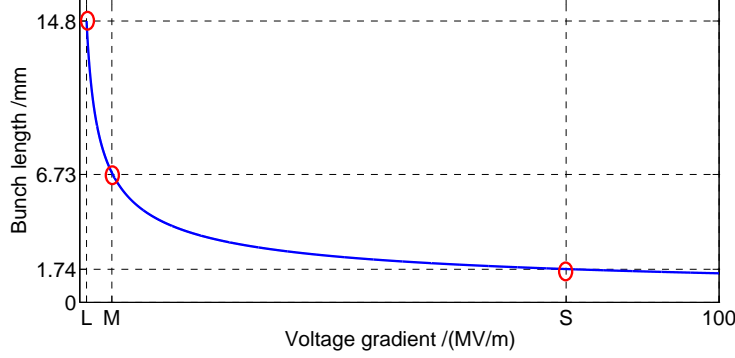


Figure 6: The bunch length in mm as a function of voltage gradient in MV/m. Three different bunches are indicated in red circles L,M,S for long , medium and short ones.

position. As $\sigma_s \propto \sqrt{\alpha/V'}$, the sum voltage gradient as a function of longitudinal position is shown in Fig.5. From Fig.5, it is clearly to see that the voltage gradient increase a lot by two sc-cavities, and three buckets are in one period. The first bucket is filled with a medium bunch, second is long , third is short,
95 and following buckets are just in the same pattern, which are indicated by the red circles in Fig.4 and Fig.5.

Actually, by applying sc-cavities in HLS II, the number of buckets increases 6 times (to 270 buckets). However, we just care about bunches at the position of original buckets. Long bunches are placed at $3m + 1$ multiples of 1.4696 m, short bunches are at $3m + 2$ multiples of 1.4696 m, and bunches at $3m$ multiples
100 of 1.4696 m are about 2.2 times shorter than long bunches length. Without Low- α mode, we can get long bunches at 14.8 mm (50 ps), short bunches at 1.74 mm (6 ps) and medium bunches at 6.73 mm (23 ps). The relation between bunch length and voltage gradient are show in Fig.6.

105 4. Dynamic Aperture

Four different groups of sextupoles per cell shown in Figure 1 are used to correct chromaticity and harmonics in HLS II. In Mode A, the operation mode in this paper, only the sextupoles in short sections are useful for chromaticity

corrections because sextupoles must be placed at locations where the dispersion
 110 function does not vanish, $\eta_x \neq 0$ [7].

Simulation for dynamic aperture by tracking 1000 turns is performed by
 ElegantRingAnalysis [8]. Fig7 shows dynamic aperture (DA) and frequency map
 for different energy spread at $\delta = 0, \pm 2\%$, which prove that DA in horizontal
 plane can reach about 40 mm. The aperture is large enough to ensure no
 115 particles loss.

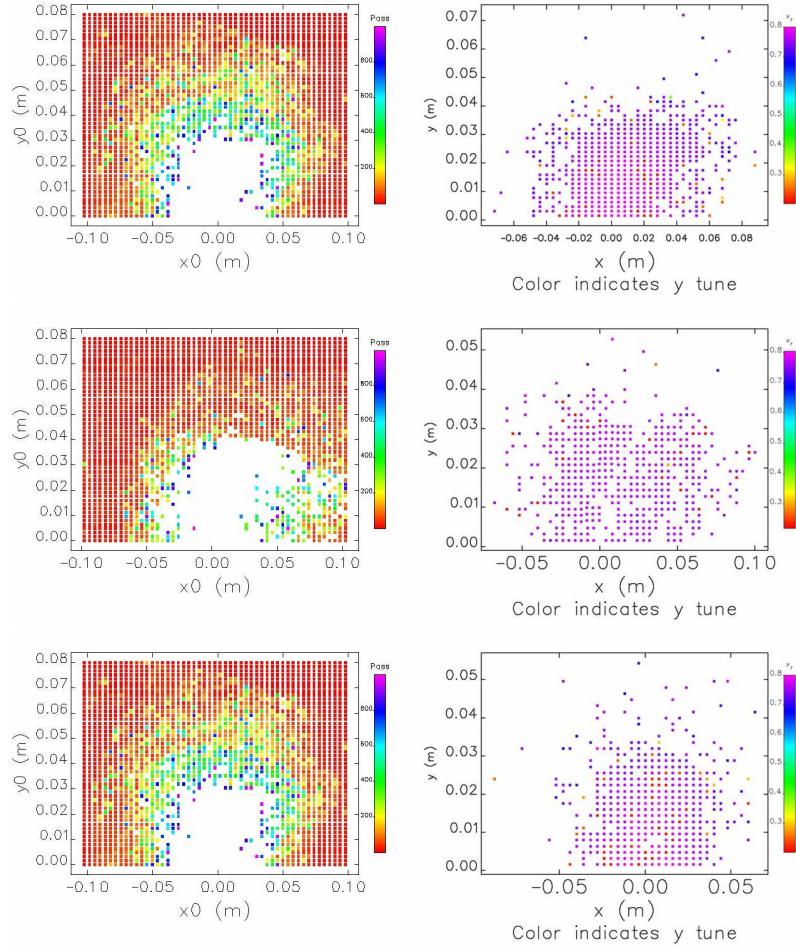


Figure 7: Dynamic aperture and frequency map analysis for $\delta = 0, \pm 2\%$

5. Tracking results

Particle tracking was performed by ELEGANT to simulate the longitudinal phase space of the new system with two sc-cavities added . The equilibrium lengths of long bunch (top) and medium (left bottom) and short (right bottom) are shown in Fig.8.

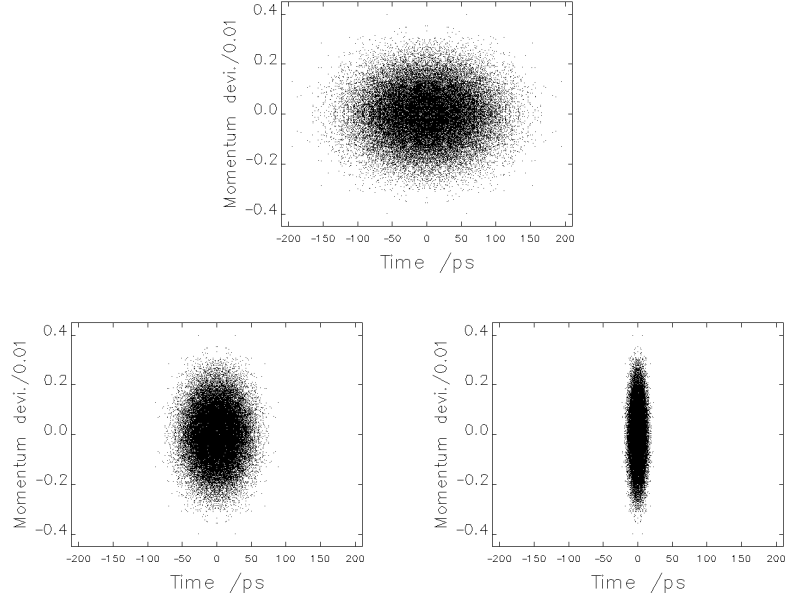


Figure 8: Results of longitudinal phase space tracking. Vertical axis indicates the momentum deviation of the bunch, and central momentum of the bunch is about 1566, horizontal axis indicates the longitudinal bunch length in picosecond

A long term of particles tracking of 1000 particles with 3 damping time (about 150000 turns) at the same initial length 20 mm was also performed by ELEGANT, to simulate the process of shortening bunches. Fig.9 indicates the length variation of three bunches, where particles start at a long, medium and short bucket location.

The data of 3 damping times can be fitted to get the equilibrium length, and the comparison to theoretical value is given in Table.5.

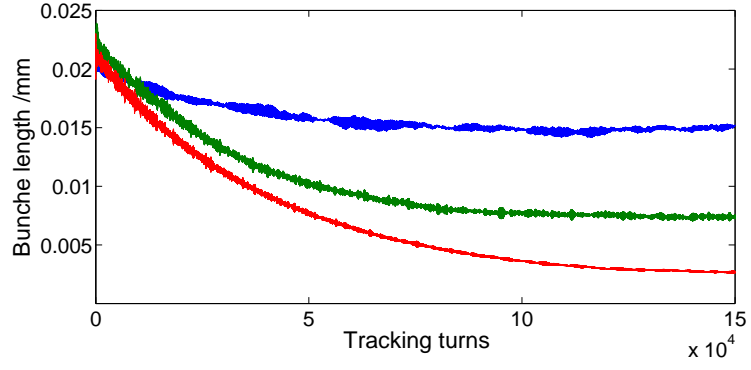


Figure 9: Results of 3 damping times tracking for length variation. Long, medium and short bunches length variation are indicated in blue, green and red. The final results of length after 3 damping times are about 15.1 mm, 7.4 mm and 2.6 mm.

Table 5: The comparison of fitting and theoretical equilibrium length

	Theoretical	Fitting length	Error
Long	14.8 mm	14.7 mm	0.7%
Medium	6.73 mm	6.59 mm	2.1%
Short	1.74 mm	1.69 mm	2.8%

6. Current Limit

The increase in the bunch length and energy spread is obvious when beam
 130 current exceeds the threshold current [7]. To avoid the bunch lengthening,
 calculation about current threshold is important.

For an electron beam of energy E_0 , a relative energy spread σ_δ , the beam
 is described by the longitudinal distribution function ϕ , which is as a sum of
 equilibrium distribution function ϕ_0 and a perturbation $\phi_1 = \hat{\phi}_1 \exp[-i\omega s/c +$
 135 $ikz]$, here k is the wave number of the perturbation. And from one-dimensional
 Vlasov equation for the distribution function[11], one can derive

$$1 = \frac{ir_0 c Z(k)}{\gamma} \int \frac{d\delta (d\phi_0/d\delta)}{\omega + ck\alpha\delta} \quad (5)$$

here $Z(k)$ is the impedance, δ is the relative energy offset of a particle and
 Gaussian distribution can be used for ϕ_0 .

Consider the CSR wakefield generated by an electron moving on a circular
 140 orbit of ρ in the middle of two parallel metal plates separated by a distance $2h$,
 the shielding parameter is given here [4]

$$\Pi = \sigma_s \rho^{1/2} / h^{3/2} \quad (6)$$

here σ_s is the bunch length. The longitudinal distribution varies at different
 bunch length.

For a long bunch, whose shielding parameter $\Pi > 3$, the CSR impedance with
 145 shielding was calculated by Warnock[12], and analysis of Eq.(6) was carried out
 for various values of a scaled current[5]

$$S = 2\sqrt{2\pi} I h / \alpha \gamma \sigma_\delta^2 I_A \sigma_s \quad (7)$$

where σ_δ is the rms relative energy spread, $I_A = 17045$ A is Alfven current. The
 beams is unstable when $S > 6/\pi$

In HLS II, shielding parameter $\Pi > 3$ corresponds the bunch length $\sigma_s > 5.8$
 150 mm, and both medium and long bunches are in the area. For bunches whose
 length is larger than 5.8 mm in HLS II, the current threshold is

$$I_{\text{th}} = I_{\alpha\sigma} \alpha \sigma_s \quad (8)$$

Here

$$I_{\alpha\sigma} = \frac{3\sqrt{2}\gamma\sigma_{\delta}^2 I_A}{2\pi^{3/2}h} \quad (9)$$

here I_{th} is the threshold of current, beam becomes unstable when the current is above I_{th} . The long bunches current limit is easy to be predicted for the threshold is proportional to momentum compaction factor and bunches length.
 155 The physic meaning of $I_{\alpha\sigma}$ could be thought as the current per momentum compaction factor per bunch length, $I_{\alpha\sigma}$ is about 112mA/mm in HLS II. As a result, the threshold current of long and medium bunches are about 34.0 mA and 15.5 mA, which are both in large value.

160 When a bunch is short, whose length is smaller than 5.8 mm in HLS II, the longitudinal wakefield requires special care in the ring for the equilibrium becomes a Haissinski distribution[13]. The bunch beam theory should be applied [5], and the current threshold should be amended as

$$I_{\text{th}} = \frac{8\pi^2 (0.5 + 0.12\Pi) \sigma_s^{7/3} [fv]_{\text{focusing}} f_{rev}}{c^2 Z_0 \rho^{1/3}} \quad (10)$$

here $Z_0 = 120\pi\Omega$, which is the impedance of free space. Bessy II also provides
 165 a empirical equation for bursting threshold calculation [10], and it agrees well with the Eq.(10). So the current threshold of short bunches is about 6.6 mA.

In comparison with low α , using superconducting RF cavities (SRF) to get short bunches could store more current in buckets. If with low- α optics to shorten the bunch at the same length above (1.74 mm), the threshold is just
 170 about 0.1 mA. To make it more clearly, we assume the threshold is $I_0 = 10$ mA at bunch length $\sigma_{z0} = 6$ mm. Then the current limit difference between two methods below 6 mm is shown in Fig.5.

Combined with Low- α , bunches can be even shorter. For the new superconducting system above, the momentum compaction factor is about $\alpha=0.0205$,
 175 with α to be adjusted to 0.000205, all bunches are expected to be even shorter by 10 times. Finally, short bunches can be about 0.174 mm (about 0.6 ps), and the total current is about 2 mA. The current is small, but it is still much better than Low- α mode. However, for users, a suitable operated α could be chosen for

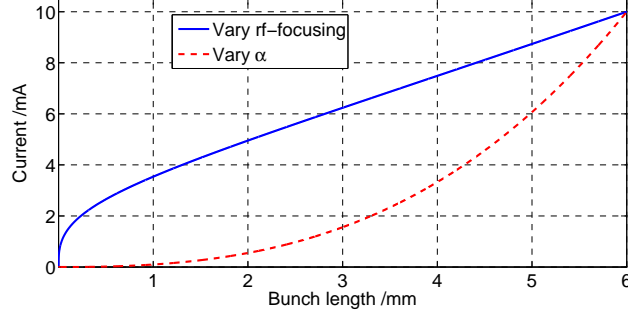


Figure 10: Current threshold when increase rf-focusing and decrease α

good current and short bunches length at the same time. Table.6 shows typical examples of expected bunches length and total current limit.

Table 6: The short bunch length and total current limit relation

α	σ_s /mm	Total current /mA
0.0205	1.74	> 300
0.0068	1.00	166
0.0024	0.6	41
0.0017	0.4	15
0.0002	0.174	2

180

7. Conclusion

A scheme is presented to operate the HLS II ring with simultaneous medium, long and short bunches. The short bunches current limit can be increased more than 60 times compared to low- α mode. The frequencies of two superconducting
185 cavities are both near 1.3 GHz, which are easy to achieve by tuning, for 1.3 GHz is a common frequency for sc-cavities [9]. And the voltages are about 2 MV, which is easy to be realized by single-cell or two-cell structure, so the HOM problems will be simplified.

8. Acknowledgements

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